Most Maximum Matchings Tree Samples

| ${ }_{\text {（empty）}} 0$ mod 7 | $1 \bmod 7$ | $2 \bmod 7$ $\ldots$ | $3 \bmod 7$ <br> －•• | $4 \bmod 7$ <br> b． |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\vdots$ ： | $\therefore$ | $\because \therefore$ | $\square 00 \therefore 0 \cdot$ |  |  | $\because \because$ |
| $\vdots \cdot \vdots$ | $+\therefore$ | $\because:$ |  | $\therefore \vdots$ | $\underset{\therefore}{\vdots}$ | $\therefore \div$ |
| $\because d$ | $\dot{A}+$ |  | $\underset{K}{x}$ | $+\cdots+\infty$ | $\therefore=\frac{5}{\vdots}$ |  |
|  | $\dot{A}+\underset{d}{ }$ |  |  |  |  |  |
| $\because \operatorname{Fax}$ | AdAd | $=\frac{1}{2}+1$ | $\cdots$ | $\dot{A N A}$ |  |  |
|  |  |  | $x=\frac{1}{x}$ |  | 栜 | $x+\cdots+\frac{Y}{i+1}$ |
|  |  | $\frac{1}{x+1}$ | $x>+\dot{Y}+\dot{C}$ |  | 成京京 |  |
| Fkthtits |  |  | $\frac{1}{i}+d$ |  |  | $i x+\frac{Y}{i}$ |
| Fthththt |  | 文 |  | thththtic |  | $x+\frac{Y}{2}+\frac{Y}{x}$ |
| Fthttatht |  | 亦市 |  |  |  | $x+\frac{\square}{i}+\cdots$ |
| ＇thathathat | thtathtathat |  | 准: 关 | （tatdaddtat |  | $\frac{y}{x+1} \frac{y}{7}$ |
|  | thathththtat |  |  | thathatandth |  |  |
|  | tattathtathtt |  | 隹道dx | thathtathatat |  | $\frac{y}{\frac{y}{7}}$ |
| Matatatatatate |  | $\begin{aligned} & \text { 进进 } \\ & \text { 进运 } \end{aligned}$ | 准書 | thtatdatdtatat | 进 |  |
| ＇tadathathathat |  | $\begin{aligned} & \text { 进 } \\ & \text { 进风进 } \end{aligned}$ | 减 |  |  | 位: |

Each column is trees of $n \bmod 7$ vertices．There are some initial exceptions then a general form in each column．Column 0 has its general form once not empty．Column 1 has its general form immediately．

Each row adds 7 further vertices in the form of a new＂C＂piece in one of the tree arms or middle． The $n=8$ case（column 1 second row）is a single C（and vertex it attaches to）．

The last initial exception is at $n=30$ in column 2 （ 5 th row 3 Cs ）．It looks like it could be the predecessor of $n=37$（ 6 th row 4 Cs ），but going up the page for successive smaller sizes in that column， the general form is one C removed from the middle every 5 rows．At $n=30$ the pattern would be remove from the middle，but the middle is already empty．

Columns 2 and 6 take a while for their patterns to show. Here are bigger examples of each to illustrate. $n=181$ is Heuberger and Wagner's sample size and is drawn in their style.


These pictures were made using a secret undocumented coordinates feature in my Graph: :Maker: : MostMaximumMatchingsTree.
http://user42.tuxfamily.org/graph-maker-other/index.html
Some completely non-visual tree building can be found in my Pari/GP vpar examples/most-maximummatchings.gp. It includes recurrences and generating functions for the number of maximum matchings. http://user42.tuxfamily.org/pari-vpar/index.html
And all this is following: Clemens Heuberger and Stephan Wagner, "The Number of Maximum Matchings in a Tree", Discrete Mathematics, volume 311, issue 21, November 2011, pages 2512-2542.
http://arxiv.org/abs/1011.6554
https://www.math.tugraz.at/~cheub/publications/max-card-matching/

